Bonus, effort, costs, market size and teams' performance

Christos Papahristodoulou

Mälardalen University/Industrial Economics

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Abstract

This paper examines the effects of a win bonus, effort, costs and team size, on the demand for talented players, the win percentage and the profits of small and big teams. Teams play a Cournot game, under the following objective functions: (i) teams maximize profits, (ii) teams maximize win percentage, (iii) the small team maximizes profit and the big win percentage, and (iv) vice versa. The effects are based on a priori selected parameter values and bounds, as well as from optimal solutions of non-linear programs, by maximizing anyone of the four win percentage formulae, derived from the respective Cournot reaction functions.

Keywords: Teams, talents, effort, win bonus, win percentage, competitive balance, profits

JEL classification: C72; L11, L83

*HST Academy/Industrial Economics, Mälardalen University, Västerås, Sweden;
christos.papahristodoulou@mdh.se Tel; +4621103176

1. Introduction

Over the last two decades, a number of researchers have applied oligopoly theory to study the performances of teams. Most of the models are based on the classical two teams' championship developed by Quirk & Fort (1992). Among the factors considered, were being the size of the teams, the effort of the players, the wage structure or the bonus system and the revenue sharing. Recently, Szymanski & Kesenne (2004), Kesenne (2007), Goossens (2007), Franck & Nüesch (2007), Papahristodoulou (2008), Dietl et al (2009) have analyzed to what extent these factors influence the win percentages or the profits of the teams.

Regarding the wage structure, Franck & Nüesch (2007), using data from the German soccer league, show that "*teams which have either a very egalitarian or a very differential pay structure are more successful on the field*" compared to teams that have "*a medium level of wage dispersion*". Papahristodoulou (2008) formulated a non-linear integer model with four teams, playing in a tournament, like the UEFA CL, under different "team-production" functions. When teams maximize points, higher wage equality seems to improve the performance of three teams, while the most efficient team of the tournament is not affected by the wage structure. When teams maximize profits, the performance depends on both the "production" technology and on wage differences. For instance, under decreasing returns, and when paying the marginal value product of their players, the most "balanced" team performs better. On the other hand, the most "unbalanced" team performs best under increasing returns to scale and egalitarian wages; in that case, the non-qualified teams did not manage to improve their performance and qualify, even if their players should receive the expected qualification bonus that UEFA pays.

Dietl et al (2009) showed that if large teams pay the highest wages (a salary cap) and small teams pay just above the salary floor, the competitive balance decreases, the profits from the small teams as well as the aggregate profits increase, but the effects on the large teams' profits are ambiguous. On the other hand, if large teams pay below cap and small teams pay exactly at the floor, the competitive balance increases, the profits from the small teams increase and the profits from the large teams decrease.

Kesenne (2007), in a two-team Cournot-tournament, assumed that the small team introduces an extra win bonus in order to improve the effort of its talents. Under plausible parameter values, and especially when the effort of its players increases, as expected by the introduction of the bonus, the profits to the small team increase and the tournament gets more balanced.

Basically, this paper is a slight modification of the Kesenne (2007) Cournot duopoly. The main difference is that the big team introduces the win bonus instead and teams compete in a mixed Cournot as well. The paper is organized as follows; in section two we present the main assumptions; in section three, teams play Cournot and maximize profits; in section four we compare our findings to Kesenne; in section five both teams maximize the win percentage, given zero profits; in section six teams play a mixed Cournot; in part one, team 1 maximizes profits and team 2 win percentage and in part two the reverse; in section 7 we concentrate on the win percentages from all models; in section 8 we compare all models based on "optimal" parameter values and in section 9 we summarize our conclusions.

2. The main assumptions

Two teams sign contracts with talents. There is a fixed amount of talents in the market, available for any demand. The number of talents each firm employs does not change during the tournament. One team pays only a fixed salary, while the other team pays both a fixed salary and a win bonus. Contracts are signed at the start of the season, before the effort of the talents was measured and the results of the tournament are known. It is assumed that team managers have a good knowledge of all relevant parameters, like effort of talents, revenues and costs. Teams maximize their win percentage or profits.

Kessene (2007) assumes that the small team (team 2) introduces the bonus to foster the effort of its players. Although there might be some small teams which introduce a win bonus, to my knowledge from the football world in many European leagues, it is more often the biggest teams, with higher economic resources, that offer a much higher win bonus. Thus, contrary to Kessene, we assume that it is the biggest team (team 1) who offers the bonus.

The win percentages are:

$$w_1 = \frac{e\,t_1}{(e\,t_1 + t_2)} \qquad (1)$$

$$w_2 = \frac{t_2}{(e\,t_1 + t_2)} \qquad (2)$$

t_1 and t_2 are the respective (homogenous) talented players, and $e \geq 1$ is the effort index required by the players of team 1. If $e t_1 = t_2$, the tournament will remain balanced, even if team 1 has less talented players than team 2.

Following Kesenne we assume that teams' cost functions are asymmetric, as below:

$$K_1 = \theta t_1 + \sigma w_1 \qquad (3)$$

$$K_2 = t_2 \qquad (4)$$

In our simulations we will assume the following parameter bounds: $0 \prec \theta \leq 1,\ 0 \leq \sigma \leq 0.5$. Thus, while team 2 pays to its talents a higher wage, equal to t_2, team 1 pays at most the same fixed salary (θ) and an "extra" win bonus (σ). This assumption might not be valid in the real world, for at least two reasons: first, even if the talented players are homogenous, bigger teams pay at least the same fixed salary as smaller ones, and also a higher bonus; second, the possibility of both $\theta \geq 1$ & $0 \prec \sigma \leq 0.5$ might be due to the fact that talented players are not perfectly homogenous, and most star players play for the biggest teams.

Precisely as Kesenne we assume the following, simple, revenue functions:

$$R_1 = m_1 w_1 \qquad (5)$$
$$R_2 = m_2 w_2 \qquad (6)$$

Team 1 is assumed to be more attractive for the public and the media (i.e. it has a higher market size) and its revenue from its winning games is higher, so that we normalize for team 2, $m_2 = 1$ and assume $m_1 \geq 1$.

Thus, since we assume that the biggest team introduces the extra bonus to its talents, it is interesting to examine if our modified assumptions would lead to less balanced tournaments and to different profits and talents, compared to Kesenne.

3. Teams play Cournot and maximize profits

The profit functions are:

$$\pi_1 = m_1 \frac{e t_1}{(e t_1 + t_2)} - \theta t_1 - \sigma \frac{e t_1}{(e t_1 + t_2)} \qquad (7)$$
$$\pi_2 = \frac{t_2}{(e t_1 + t_2)} - t_2 \qquad (8)$$

Differentiate (7) and (8) w. r. t. t_1 and t_2 respectively and solve we obtain the teams' talents demands:

$$t_1 = \frac{e(\sigma^2 - 2\sigma m_1 + m_1^2)}{(-\theta + e\sigma - e m_1)^2} \qquad (9)$$
$$t_2 = -\frac{e\theta(\sigma - m_1)}{(\theta - e\sigma + e m_1)^2} \qquad (10)$$

Setting (9) and (10) in the profit functions (7) and (8) we obtain:

$$\pi_1 = -\frac{e^2(\sigma - m_1)^3}{(\theta - e\sigma + e m_1)^2} \qquad (11)$$
$$\pi_2 = \frac{\theta^2}{(\theta - e\sigma + e m_1)^2} \qquad (12)$$

Finally, setting (9) and (10) in (1) and (2) we obtain:

$$w_1 = \frac{e(\sigma - m_1)}{e\sigma - em_1 - \theta} \quad (13)$$

$$w_2 = \frac{\theta}{\theta - e\sigma + em_1} \quad (14)$$

From (13) and (14) it is clear that the tournament is completely balanced, if $\theta = e(m_1 - \sigma)$, team 1 wins more if $\theta < e(m_1 - \sigma)$ and team 2 wins more if $\theta > e(m_1 - \sigma)$.

Regarding talents, the simplified ratio is $\frac{t_1}{t_2} = \frac{(m_1 - \sigma)}{\theta}$, i.e. independent from effort. Moreover, effort influences the overall number of talents, i.e. both the numerator and the denominator and the balance of the tournament. For instance, if $e = 1, \theta = 1, \sigma = 0.5, m_1 = 1.4$ then, $t_1 = t_2 = 0.25$ and the tournament is balanced; but if we change the effort to $e = 1.4$, and use the initial formulae (9) and (10), both values decrease to $t_1 = t_2 = 0.243$. Thus, despite the fact that the talents ratio is unchanged, the tournament turns unbalanced.

Finally regarding profits, it is also unclear whether (11) is larger or lower than (12). For instance, it is possible that $\pi_1 < \pi_2$, if the following conditions *(a)* apply:

$$m_1 < 1.5 \,\&\, -1 + m_1 < \sigma < 0.5 \,\&\, \sqrt{-\sigma^3 + 3\sigma^2 m_1 - 3\sigma m_1^2 + m_1^3} < \theta \le 1 \,\&\, 1 \le e < \sqrt{-\frac{1\theta^2}{(\sigma - 1m_1)^3}} \quad (a)$$

Table 1 summarizes the effects of the four parameters on profits, talents and the win percentages. The sign of the derivatives is based on the following, rather broad, parameter bounds: $e \ge 1, 0 \prec \theta \le 1, 0 \prec \sigma \prec 0.5, m_1 \succ 1.$

The effects on the profits and the win percentage (the first two parts of the Table) are similar. As expected, higher effort and higher market size increase the win percentage and the profits of team 1 and decrease the win percentage and profit of team 2. On the other hand, higher values in fixed salary (θ) and bonus (σ), affect team 1 negatively and team 2 positively.

The effects on teams' talents (and especially for the smaller team 2) are more complex though. In general, effort can have a positive effect on both teams' talents, if the same following conditions *(b)* are valid:

$$1 < m_1 < 1.5 \,\&\, -1 + m_1 < \sigma < 0.5 \,\&\, -1\sigma + m_1 < \theta \le 1 \,\&\, 1 \le e < -\frac{1\theta}{\sigma - 1m_1} \quad (b)$$

Notice that this condition requires an upper limit to effort equal to 2, provided that: *(i)* the market size to team 1 is a little higher than 1; *(ii)* its fixed salary is close to 1; *(iii)* its bonus is close to 0.5. On the other hand, by violating the last two conditions, using for instance $e = 1.1,\ \theta = .95,\ \sigma = .3,\ m_1 = 1.4$, the respective talent values are *0.2853* and *0.2464* and by just increasing effort to *e = 1.6*, both values are reduced to *0.2636* and *0.2277* respectively.

Table 1: Effects from the profit maximization

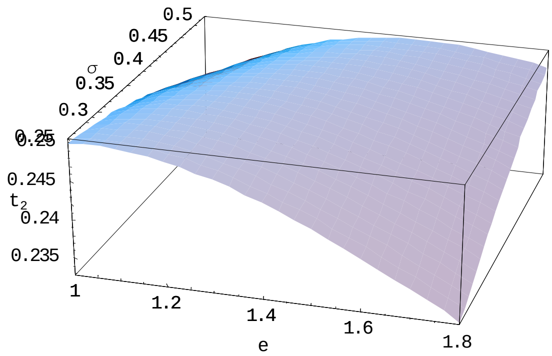
Derivative	Sign	Derivative	Sign
$\frac{\partial w_1}{\partial e} = \frac{\theta(-\sigma + m_1)}{(\theta - e\sigma + em_1)^2}$	+	$\frac{\partial w_2}{\partial e} = \frac{\theta(\sigma - m_1)}{(\theta - e\sigma + em_1)^2}$	-
$\frac{\partial w_1}{\partial m_1} = \frac{e\theta}{(\theta - e\sigma + em_1)^2}$	+	$\frac{\partial w_2}{\partial m_1} = -\frac{e\theta}{(\theta - e\sigma + em_1)^2}$	-
$\frac{\partial w_1}{\partial \theta} = \frac{e(\sigma - m_1)}{(\theta - e\sigma + em_1)^2}$	-	$\frac{\partial w_2}{\partial \theta} = \frac{e(-\sigma + m_1)}{(\theta - e\sigma + em_1)^2}$	+
$\frac{\partial w_1}{\partial \sigma} = -\frac{e\theta}{(\theta - e\sigma + em_1)^2}$	-	$\frac{\partial w_2}{\partial \sigma} = \frac{e\theta}{(\theta - e\sigma + em_1)^2}$	+
$\frac{\partial \pi_1}{\partial e} = -\frac{2e\theta(\sigma - m_1)^3}{(\theta - e\sigma + em_1)^3}$	+	$\frac{\partial \pi_2}{\partial e} = \frac{2\theta^2(\sigma - m_1)}{(\theta - e\sigma + em_1)^3}$	-
$\frac{\partial \pi_1}{\partial m_1} = \frac{e^2(\sigma - m_1)^2(3\theta - e\sigma + em_1)}{(\theta - e\sigma + em_1)^3}$	+	$\frac{\partial \pi_2}{\partial m_1} = -\frac{2e\theta^2}{(\theta - e\sigma + em_1)^3}$	-
$\frac{\partial \pi_1}{\partial \theta} = \frac{2e^2(\sigma - m_1)^3}{(\theta - e\sigma + em_1)^3}$	-	$\frac{\partial \pi_2}{\partial \theta} = \frac{2e\theta(-\sigma + m_1)}{(\theta - e\sigma + em_1)^3}$	+
$\frac{\partial \pi_1}{\partial \sigma} = \frac{e^2(\sigma - m_1)^2(-3\theta + e\sigma - em_1)}{(\theta - e\sigma + em_1)^3}$	-	$\frac{\partial \pi_2}{\partial \sigma} = \frac{2e\theta^2}{(\theta - e\sigma + em_1)^3}$	+
$\frac{\partial t_1}{\partial e} = \frac{(\sigma - m_1)^2(\theta + e\sigma - em_1)}{(\theta - e\sigma + em_1)^3}$	?	$\frac{\partial t_2}{\partial e} = -\frac{\theta(\sigma - m_1)(\theta + e\sigma - em_1)}{(\theta - e\sigma + em_1)^3}$	?
$\frac{\partial t_1}{\partial m_1} = \frac{2e\theta(-\sigma + m_1)}{(\theta - e\sigma + em_1)^3}$	+	$\frac{\partial t_2}{\partial m_1} = \frac{e\theta(\theta + e\sigma - em_1)}{(\theta - e\sigma + em_1)^3}$	?
$\frac{\partial t_1}{\partial \theta} = -\frac{2e(\sigma - m_1)^2}{(\theta - e\sigma + em_1)^3}$	-	$\frac{\partial t_2}{\partial \theta} = \frac{e(\sigma - m_1)(\theta + e\sigma - em_1)}{(\theta - e\sigma + em_1)^3}$	?
$\frac{\partial t_1}{\partial \sigma} = \frac{2e\theta(\sigma - m_1)}{(\theta - e\sigma + em_1)^3}$	-	$\frac{\partial t_2}{\partial \sigma} = -\frac{e\theta(\theta + e\sigma - em_1)}{(\theta - e\sigma + em_1)^3}$	?

As expected, the higher the market size, the more talents are demanded by the big team. On the other hand, the effect on t_2 is ambiguous. It will be positive, under exactly the same conditions *(a)* above. For instance, if team 1 pays a rather high bonus, ($\sigma = 0.3$) and a fixed salary, $\theta = 0.95$, the effort from its talents is not that high, (e = *1.1*), and its market size is not higher than *1.16*, team 2 will demand more talents than team 1!

The demand for talents t_1 is lower when the fixed salary or the bonus increases, while these effects on t_2 are again ambiguous. For instance, the effects of θ and σ on t_2 can be <u>negative</u> under exactly the same conditions as before, *(a)* or *(b)*.

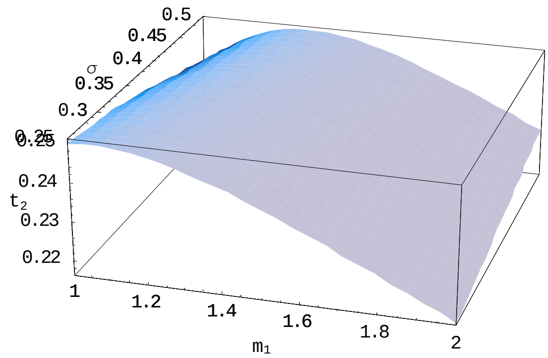
Below we plot t_2, against e and σ, given the following parameters, $m_1 = 1.1$, and $\theta = 0.9$. It is clear that talents for the small team are decreasing for simultaneously higher values of e and low values of σ. On the other hand, for higher values of σ and e, talents are increasing and reach a maximum at $e = 1.5$, before they decrease again.

Graph 1: The demand for t_2 as a function of efforts and bonus



In the second graph we plot t_2 against two other parameters, m_1 and σ, given $e = 1.1$ and $\theta = 0.9$. It is clear that, if team 1 pays a relatively high win bonus and its market is not high, (i.e. $m_1 < 1.3$) the talents of team 2 will increase.

Graph 2: The demand for t_2 as a function of market size and bonus



Thus, while the largest team 1 demands less talents when it pays high fixed salary (θ), or extra win bonus (σ), it is possible that team 2 will demand more talents instead! On the other hand, a high market size of team 1 results to more talents, more wins and more profits.

4. A comparison to Kesenne results

As mentioned earlier, our modified assumptions are expected to lead to less competitive balance, compared to Kesenne (2007), and to different values in profits and talents. In Table 2 we show our own and Kesenne results, using the same five pair of parameters selected by Kesenne. In all simulations, the biggest team has a double size (i.e. $m_1 = 2$).

Table 2: Own and Kesenne simulation results ($m_1 = 2$)

	w_1	w_2	t_1	t_2	$t_1 + t_2$ normalized	π_1	π_2	$\pi_1 + \pi_2$ normalized	*Results*
$\theta = 1$	.6666	.3333	.4444	.2222	66	.8889	.1111	100	*Own*
$\sigma = 0$	.6666	.3333	.4444	.2222	66	.8889	.1111	100	*Kesenne*
$e = 1$									
$\theta = 1$	.7118	.2882	.3897	.2051	59	.9630	.0830	105	*Own*
$\sigma = .1$	.6309	.3691	.4657	.2096	68	.7961	.1226	92	*Kesenne*
$e = 1.3$									
$\theta = 1$	.6552	.3448	.4292	.2259	66	.8155	.1189	93	*Own*
$\sigma = .1$	.6897	.3103	.4281	.1926	62	.9512	.0867	104	*Kesenne*
$e = 1$									
$\theta = .85$	.7440	.2560	.4258	.1905	62	1.0516	.0655	112	*Own*
$\sigma = .1$	.5923	.4077	.4829	.2557	74	.7017	.1496	85	*Kesenne*
$e = 1.3$									
$\theta = .85$	.6909	.3091	.4773	.2135	69	.9069	.0955	100	*Own*
$\sigma = .1$	.6538	.3462	.4527	.2396	69	.8550	.1078	96	*Kesenne*
$e = 1$									

In the first raw, with the same fixed salary, without bonus and no extra effort, both results are identical. In three cases, our results reveal indeed a lower competitive balance compared to Kesenne.

The only case in which the Kesenne model leads to lower competitive balance is in the third raw, where despite the fact that the bonus is being paid, in addition to the same fixed salary, the effort remains unchanged. If the small team introduces the bonus its win percentage is *0.3108*, while if the big team introduces it, the small team improves its win percentage to *0.3448*! We conclude that if the bonus does not improve the effort, the competitive balance will be higher when the big team introduces it.

The most extreme case is found in the fourth raw (lower fixed salary, some bonus and higher effort), where in the Kesenne model the competitive balance improves strongly while in our model it is reduced significantly. In that case, the competitive balance will be higher when the small team introduces it.

Regarding the normalized values and the distribution of talents to the teams, the results are unclear. It seems though that the Kesenne results for team 1 are higher compared to ours. Our results in the second raw (with equal fixed salary, a small bonus and a rather high effort) provide the lowest total value and also the most evenly distribution.

Finally, our total profits seem to be higher compared to Kesenne. The only case in which the Kesenne model leads to higher overall profits is in the third raw. In the same case, the profits to team 2 are higher according to our model. The highest difference in profits is found in the fourth raw, where the profits to team 2 are the highest according to Kesenne and the lowest according to our model.

5. Teams play Cournot and maximize win percentages

We turn now to win maximization problem, subject to zero profits. The zero-profits constraint is obviously not very realistic, but it simplifies significantly the extremely complex derivatives that would occur if we assumed non-negative profits and derived the Kuhn-Tucker conditions.

We continue assuming that team 1 introduces effort, i.e. equations (1) and (2) are the same. Since the mathematical complexities increase dramatically with many parameters, we assume that the cost functions are now simpler and similar to both teams:

$$C_1=c_1t_1 \qquad C_2=c_2t_2 \qquad (3)', (4)'$$

In our simulations later we will assume that $c_1 \geq c_2$.

The revenue functions are also similar to both teams and quadratic in the win percentage:

$$R_1=m_1w_1-\beta\left(w_1\right)^2 \qquad R_2=m_2w_2-\beta\left(w_2\right)^2 \qquad (5)', (6)'$$

Quadratic revenue functions reduce revenues, when excessive winning make matches less exciting and does not attract a huge public. Moreover, for rather high m- and low β-values, the fall in revenues is very modest. To simplify our derivatives, we use the same β-parameter to both teams and assume that $1 > \beta \succ 0,\ m_1 \geq 1,\ m_2 \geq \beta,\ m_1 \geq m_2$.

The zero profits conditions are valid when average revenues equal average costs, given by the following conditions.

$$\frac{m_1 w_1 - b(w_1)^2}{t_1} = c_1$$
$$\frac{m_2 w_2 - b(w_2)^2}{t_2} = c_2 \qquad (15), (16)$$

Both teams maximize:

$$L = \frac{et_1}{et_1 + t_2} + \lambda\left(\frac{m_1 \frac{et_1}{et_1+t_2} - \beta\left(\frac{et_1}{et_1+t_2}\right)^2}{t_1} - c_1\right) \qquad (17)$$

$$M = \frac{t_2}{et_1 + t_2} + \mu\left(\frac{m_2 \frac{t_2}{et_1+t_2} - \beta\left(\frac{t_2}{et_1+t_2}\right)^2}{t_2} - c_2\right) \qquad (18)$$

The simplified first order conditions are:

$$\frac{e(t_2(-e\beta\lambda - e\lambda m_1 + t_2) + e t_1 (e\beta\lambda - e\lambda m_1 + t_2))}{(e t_1 + t_2)^3} = 0 \qquad (19)$$

$$-c_1 + \frac{e(e(-\beta + m_1)\, t_1 + m_1 t_2)}{(e t_1 + t_2)^2} = 0 \qquad (20)$$

$$\frac{e^2 t_1^2 - e t_1 (\beta\mu + \mu m_2 - t_2) + \mu(\beta - m_2)\, t_2}{(e t_1 + t_2)^3} = 0 \qquad (21)$$

$$\frac{-\beta t_2 + m_2 (e t_1 + t_2) - c_2 (e t_1 + t_2)^2}{(e t_1 + t_2)^2} = 0 \qquad (22)$$

Solving the system and simplifying we find:

$$t_1 = \frac{(e c_2 m_1 + c_1 (\beta - m_2))(-\beta + m_1 + m_2)}{\beta (c_1 + e c_2)^2} \qquad (23)$$

$$t_2 = -\frac{e(\beta - m_1 - m_2)(e c_2 (\beta - m_1) + c_1 m_2)}{\beta (c_1 + e c_2)^2} \qquad (24)$$

(23) is strictly positive, as long as $c_2 \succ \frac{c_1 - \beta c_1}{e}$, and (24) is strictly positive, as long as $c_2 \prec -\frac{c_1}{e\beta - \beta m_1}$.

Set (23) and (24) in (1) and (2) and simplifying, we find:

$$w_1 = \frac{e c_2 m_1 + c_1 (\beta - m_2)}{\beta (c_1 + e c_2)} \qquad (25)$$

$$w_2 = \frac{e c_2 (\beta - m_1) + c_1 m_2}{\beta (c_1 + e c_2)} \qquad (26)$$

(25) is strictly positive, as long as $c_2 \succ \frac{-\beta c_1 + c_1 m_2}{e m_1}$, and (26) is strictly positive, as long as $c_2 \prec -\frac{c_1 m_2}{e\beta - e m_1}$.

The sign of the partial derivatives of (23) - (26) are shown in Table 3. In the first half of the table we show the sign of the win percentage derivatives, given some broad parameter bounds. The effects on the win percentages are similar to Table 1. In both models, effort and market size of team 1 affect the winning performance of team 1 positively, while the cost c_1, which is quite similar to θ (mainly) and also σ in the first model, affects it negatively. The β–effect[1] depends on the relationship between c_2 and $\frac{c_1 m_2}{e m_1}$. For instance, for the normalized values $c_2 = m_2 = 1$, the parameter β will have a positive winning effect on team 1, if $c_1 > e m_1$.

Table 3: Effects from win maximization

$e \geq 1,\ 1 \succ \beta \succ 0,\ (c_1, c_2) \succ 0,\ m_1 \geq 1,\ m_1 \geq m_2,\ m_2 \geq \beta$				$(\beta = 0.5,\ m_2 = 1,\ c_2 = 1,\ m_1 = 1.5,\ e = 1.1,\ c_1 \geq 1)$ $(\beta = 0.5,\ m_2 = 1,\ c_2 = 1,\ c_1 = 1.2,\ e = 1.3,\ m_1 \geq 1)$			
$\frac{\partial w_1}{\partial c_1}$	-	$\frac{\partial w_2}{\partial c_1}$	+	$\frac{\partial t_1}{\partial c_1}$	$(c_1 \succ 7.7)$ (-)	$\frac{\partial t_2}{\partial c_1}$	$(c_1 \prec 3.3)$ (+)
$\frac{\partial w_1}{\partial c_2}$	+	$\frac{\partial w_2}{\partial c_2}$	-	$\frac{\partial t_1}{\partial c_2}$	(+) $(m_1 \prec 12)$	$\frac{\partial t_2}{\partial c_2}$	(-) $(m_1 \succ 24.5)$
$\frac{\partial w_1}{\partial m_1}$	+	$\frac{\partial w_2}{\partial m_1}$	-	$\frac{\partial t_1}{\partial m_1}$	$(c_1 \prec 7.7)$ (+)	$\frac{\partial t_2}{\partial m_1}$	$(c_1 \succ 3.3)$ (-)
$\frac{\partial w_1}{\partial m_2}$	-	$\frac{\partial w_2}{\partial m_2}$	+	$\frac{\partial t_1}{\partial m_2}$	(-) $(m_1 \succ 12)$	$\frac{\partial t_2}{\partial m_2}$	(+) $(m_1 \prec 24.5)$
$\frac{\partial w_1}{\partial \beta}$	+, *if* $c_2 \prec \frac{c_1 m_2}{e m_1}$	$\frac{\partial w_2}{\partial \beta}$	+, *if* $c_2 \succ \frac{c_1 m_2}{e m_1}$	$\frac{\partial t_1}{\partial \beta}$	$(c_1 \succ 1.8333)$ (-)	$\frac{\partial t_2}{\partial \beta}$	$(c_1 \prec 1.54)$ $(m_1 \succ 1.0453)$
$\frac{\partial w_1}{\partial e}$	+	$\frac{\partial w_2}{\partial e}$	-	$\frac{\partial t_1}{\partial e}$	(+) $(m_1 \prec 12)$	$\frac{\partial t_2}{\partial e}$	$(c_1 \succ 3.3)$ (-)

In the second half of the table, we show the partial derivatives of talents. Most of these derivatives are very complex and non-linear. In order to analyze under which costs and market size conditions the derivatives for both teams are positive, we assumed first the following specific values: $\beta = 0.5,\ m_2 = 1,\ c_2 = 1,\ m_1 = 1.5,\ e = 1.1$. Then we solved for the cost to team 1, satisfying the bound $c_1 \geq 1$. The minimum or maximum bounds of c_1 required for positive derivatives, are given in the first raw.

[1] Since we assumed the same β-parameter for both teams, the change of the value can be the result of any team.

We repeated for the market size of team 1, satisfying the bound $m_1 \geq 1$ and also setting $\beta = 0.5,\ m_2 = 1,\ c_2 = 1,\ c_1 = 1.2,\ e = 1.3.$ The minimum or maximum bounds of m_1 required for positive derivatives, are given in the second raw. The (+) signs satisfy the minimum bounds, $c_1 \geq 1$ or $m_1 \geq 1$, while the (-) signs reject the existence of positive derivatives for these specific values and bounds.

The effects of both e and c_2 on t_1 are identical. Both are positive, as long as $c_1 \geq 1$, or as long as $12 \succ m_1$. Moreover, due to non-linearity in the derivatives, extremely large market size of team 1 would turn these effects to negative ones! The effect of m_1 on t_1 is positive, as long as $7.7 \succ c_1$, or as long as $m_1 \geq 1$, and turns negative for c_1 above 7.7. The effect of c_1 on t_1 is opposite to the effect of m_1, i.e. it is negative, unless the costs are extremely high (above 7.7); it is never positive with the second set of parameters. The effect of m_2 on t_1 is never positive with the first set of parameters; it is positive though, if the market size of team 1 is extremely high, i.e. if it is more than 12 times larger than the market size of team 2. Finally, the effect of β on t_1 is positive if team 1 pays at least 83.3% higher costs and always negative, irrespectively how larger the market size of team 1 is.

The effects of both e and m_1 on t_2 are also identical. Both are positive, as long as $c_1 \succ 3.3$, and both are negative as long as $m_1 \geq 1$. The effect of m_2 on t_2 is positive, as long as $c_1 \geq 1$, or as long as $24.5 \succ m_1 \geq 1$. The effect of c_1 on t_2 is positive, if the costs to team 1 are below the upper limit of 3.3, or if $m_1 \geq 1$. Also the effect of β on t_2 is positive if the cost limits are $1.54 \succ c_1 \geq 1$, or if $m_1 \geq 1.0453$. Finally, the effect of c_2 on t_2 is negative, if $c_1 \geq 1$ or if $m_1 \prec 24.5$.

Notice that the minimum or maximum values of c_1 and m_1 required for positive effects on t_1 are not equal to the respective maximum or minimum values required for positive effects on t_2. For instance, the effects of m_1 on both t_2 and t_1 are positive for $c_1 \succ 3.3$, respectively $c_1 \prec 7.7$, such as $c_1 = 4$; but, for $c_1 = 8$, the effect on t_2 remains positive, while the effect on t_1 turns negative.

Most of the effects on t_1 and t_2 seem to be consistent with those in Table 1, but the restrictiveness in these two models differs. For instance, m_1 affects t_1 positively in both models. Moreover, while in the profit maximization that is clear, in the win maximization model it is also required that $c_1 \prec 7.7$. Similarly, while in the profit maximization the effect of θ, is clearly negative, in the win maximization model the corresponding c_1 will have a negative effect on t_1 only if $c_1 \prec 7.7$. On the other hand, in the win maximization model, the effort effect on t_1 is positive under less restrictive conditions, like those set in Table 3. The profit maximization conditions are much stronger though, because, in addition to $m_1 = 1.5,\ e = 1.1$, two more strong conditions are required, namely: $1 \geq \theta \succ 0.99$ & $\sigma = 0.6$. For instance, if we limit the bonus parameter to its upper bound 0.5, the effort effect on t_1 turns negative.

If we turn to the effects on t_2, we found earlier, Table 1, that the derivatives are ambiguous. On the other hand, in the win maximization model, the effects of e and m_1 are negative under more plausible conditions, such as $m_1 \geq 1$, or $c_1 \prec 3.3$, and the effect of c_1 is positive under exactly the same conditions. The respective θ effect on t_2, in the profit maximization model, would be positive, given the same parameters $m_1 = 1.5$, $e = 1.1$ and also if $\sigma < 0.59$ (i.e. violating the upper limits).

6. Teams play mixed Cournot: one maximizes profits and the other maximizes win percentage

In football, the objective functions of teams might differ, despite the fact that they compete in the same tournament. Sometimes, when the owner of a team has maximized the win percentage over a period of years, by scarifying profits, or even incurring losses, he might change his objective and maximize profits instead. Similarly, owners who have maximized profits over a certain period of years might be forced by their public to shift to win maximization.

Two examples from the Italian Serie A are AC Milan and FC Inter. The owner of AC Milan, Silvio Berlusconi, when he took over the team, his main objective was to win everything (and entertain as well), without bothering about profits. The great success of his team had two effects; first, it incurred considerable losses and second, it increased the popularity of its owner. When Silvio Berlusconi was elected Italy's prime minister, AC Milan lost a lot of its win percentage, because its owner changed his objective into profit maximization, (in fact, loss minimization). Instead of investing in new players, AC Milan started selling its stars, like Kaká in summer of 2009[2]. On the other hand, his fierce competitor, Massimo Moratti, the owner of FC Inter, who has not won as much as AC Milan, (at least in European competitions), was spurred to maximize the win percentage, by buying expensive players and paying less attention to profits.

We turn therefore to the mixed Cournot game, in which the teams have different objective functions. In case (a), the small team maximizes profits while the big team maximizes win percentage and in case (b) just the opposite.

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[2] "*I am concerned about the salary. The fees of the players are ineligible. We should get one day to fix a salary cap, like the US. I have spoken with Platini. I am the first to 'jump the bench'? I paid 10 billion lire for Gullit and achieved 15 in advertising for my TV. It was a deal. And now the salaries in Milan are lower than other clubs, because here it is a society that treats people in a certain way. Is my opinion an attack on the Real? I do not criticize the Spaniards, the phenomenon is general. Also a cut by 50 percent, the current salary would be insane.* "*Silvio Berlusconi (Gazzetta dello Sport, August, 19, 2009)*

(a) Team 2 maximizes profits and team 1 maximizes win percentage

We need the profit function (8), modified slightly by multiplying the normalized wages paid to its talents t_2 with a wage parameter $\gamma > 0$, and also the win percentage function (17).

From the respective first order conditions we obtain:

$$t_1 = \frac{e\gamma m_1^2}{(e\beta\gamma + c_1)^2} \tag{27}$$

$$t_2 = \frac{e(c_1 + e\gamma(\beta - m_1))\, m_1}{(e\beta\gamma + c_1)^2} \tag{28}$$

Thus, the win percentages are:

$$w_1 = \frac{e\gamma m_1}{e\beta\gamma + c_1} \tag{29}$$

$$w_2 = \frac{c_1 + e\gamma(\beta - m_1)}{e\beta\gamma + c_1} \tag{30}$$

The profit for team 2 is:

$$\pi_2 = \frac{(c_1 + \gamma e(\beta - m_1))^2}{(\gamma e\beta + c_1)^2} \tag{31}$$

Assuming the following bounds in parameters, $e \geq 1,\ c_1 \succ 0,\ 1 \succ (\beta, \gamma) \succ 0,\ m_1 \geq 1$, it is easy to show that the sum of (27) and (28) is strictly positive and (27) is larger than (28).

Given the bounds above, we show in Table 4a the sign of derivatives, and under which additional conditions[3] they are positive.

It is clear that e, m_1 and γ have a positive effect on w_1, while β and c_1 have a negative effect. Thus, when team 1 attracts a huge public, i.e. when m_1 is high or β is low, its win percentage increases, while when its revenue decreases, either by lower m- or higher β-values, its win percentage falls[4]. Similarly, m_1 has a positive effect on t_1 while β and c_1 have a negative effect. Also the effects of e and γ on t_1 are positive, as

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[3] To simplify these additional conditions, we assumed $\beta = 0.5$. The conditions shown in the Table are the simplest ones, because there are also more complex conditions for positive derivatives.

[4] The interpretation of these derivatives is that team 1 can't win extensively with less home public (smaller market size), or when many of its supporters find it less exciting (higher β). Perhaps, both home public and win percentages are endogenous, and when the team does not win so extensively anymore, very often, a large part of the home public is disappointed and does not follow its matches.

long as $c_1 > 0.5e\gamma$. Thus, team 1 can afford to pay high costs and still demand more talents, when the product of effort, fixed salary paid by team 2 and β is lower than c_1.

Table 4a: Effects from mixed Cournot (team 2 maximizes profits)

$\frac{\partial w_1}{\partial e} = \frac{\gamma c_1 m_1}{(e\beta\gamma + c_1)^2}$	+	$\frac{\partial t_1}{\partial e} = \frac{\gamma(-e\beta\gamma + c_1) m_1^2}{(e\beta\gamma + c_1)^3}$	$c_1 > 0.5e\gamma$
$\frac{\partial w_1}{\partial \gamma} = \frac{e c_1 m_1}{(e\beta\gamma + c_1)^2}$	+	$\frac{\partial t_1}{\partial \gamma} = \frac{e(-e\beta\gamma + c_1) m_1^2}{(e\beta\gamma + c_1)^3}$	$c_1 > 0.5e\gamma$
$\frac{\partial w_1}{\partial \beta} = -\frac{e^2\gamma^2 m_1}{(e\beta\gamma + c_1)^2}$	-	$\frac{\partial t_1}{\partial \beta} = -\frac{2e^2\gamma^2 m_1^2}{(e\beta\gamma + c_1)^3}$	-
$\frac{\partial w_1}{\partial m_1} = \frac{e\gamma}{e\beta\gamma + c_1}$	+	$\frac{\partial t_1}{\partial m_1} = \frac{2e\gamma m_1}{(e\beta\gamma + c_1)^2}$	+
$\frac{\partial w_1}{\partial c_1} = -\frac{e\gamma m_1}{(e\beta\gamma + c_1)^2}$	-	$\frac{\partial t_1}{\partial c_1} = -\frac{2e\gamma m_1^2}{(e\beta\gamma + c_1)^3}$	-
$\frac{\partial w_2}{\partial e} = -\frac{\gamma c_1 m_1}{(e\beta\gamma + c_1)^2}$	-	$\frac{\partial t_2}{\partial e} = \frac{c_1(c_1 + e\gamma(\beta - 2m_1)) m_1}{(e\beta\gamma + c_1)^3}$	$c_1 > -0.5e\gamma + 2e\gamma m_1$
$\frac{\partial w_2}{\partial \gamma} = -\frac{e c_1 m_1}{(e\beta\gamma + c_1)^2}$	-	$\frac{\partial t_2}{\partial \gamma} = \frac{e^2 m_1(e\beta\gamma(-\beta + m_1) - c_1(\beta + m_1))}{(e\beta\gamma + c_1)^3}$	$c_1 < \frac{-0.25e\gamma + 0.5e\gamma m_1}{0.5 + m_1}$
$\frac{\partial w_2}{\partial \beta} = \frac{e^2\gamma^2 m_1}{(e\beta\gamma + c_1)^2}$	+	$\frac{\partial t_2}{\partial \beta} = -\frac{e^2\gamma(c_1 + e\gamma(\beta - 2m_1)) m_1}{(e\beta\gamma + c_1)^3}$	$c_1 < -0.5e\gamma + 2e\gamma m_1$
$\frac{\partial w_2}{\partial m_1} = -\frac{e\gamma}{e\beta\gamma + c_1}$	-	$\frac{\partial t_2}{\partial m_1} = \frac{e(c_1 + e\gamma(\beta - 2m_1))}{(e\beta\gamma + c_1)^2}$	$c_1 > -0.5e\gamma + 2e\gamma m_1$
$\frac{\partial w_2}{\partial c_1} = \frac{e\gamma m_1}{(e\beta\gamma + c_1)^2}$	+	$\frac{\partial t_2}{\partial c_1} = -\frac{e(c_1 + e\gamma(\beta - 2m_1)) m_1}{(e\beta\gamma + c_1)^3}$	$c_1 < -0.5e\gamma + 2e\gamma m_1$
$\frac{\partial \pi_2}{\partial e} = -\frac{2\gamma c_1(c_1 + e\gamma(\beta - m_1)) m_1}{(e\beta\gamma + c_1)^3}$			$c_1 \prec -0.5e\gamma + e\gamma m_1$
$\frac{\partial \pi_2}{\partial \gamma} = -\frac{2e c_1(c_1 + e\gamma(\beta - m_1)) m_1}{(e\beta\gamma + c_1)^3}$			$c_1 \prec -0.5e\gamma + e\gamma m_1$
$\frac{\partial \pi_2}{\partial \beta} = \frac{2e^2\gamma^2(c_1 + e\gamma(\beta - m_1)) m_1}{(e\beta\gamma + c_1)^3}$			$c_1 \succ -0.5e\gamma + e\gamma m_1$
$\frac{\partial \pi_2}{\partial m_1} = -\frac{2e\gamma(c_1 + e\gamma(\beta - m_1))}{(e\beta\gamma + c_1)^2}$			$c_1 \prec -0.5e\gamma + e\gamma m_1$
$\frac{\partial \pi_2}{\partial c_1} = \frac{2e\gamma(c_1 + e\gamma(\beta - m_1)) m_1}{(e\beta\gamma + c_1)^3}$			$c_1 \succ -0.5e\gamma + e\gamma m_1$

With regard to t_2, the effects are ambiguous. For instance the effects of e and m_1 can be positive, as long as $c_1 \succ -0.5e\gamma + 2e\gamma m_1$, while the effects of β and c_1 are negative for the same condition and finally γ will have a positive effect on t_2, for a very small cost interval.

The effects on π_2 differ. Effort, market size and γ influence π_2 positively, as long as $c_1 \prec -0.5e\gamma + e\gamma m_1$. Also β and c_1 will influence π_2 positively, when the opposite inequality is valid. But, checking the conditions for which both π_2 and t_2 are positive, we conclude that the range of parameters differs. For instance, given $\gamma = 1$ and $m_1 = 2$, the effort effect on π_2 is positive if, $e > 0.6667c_1$ and $c_1 > 1.5$; the same effect on t_2 is positive if either, (i) $e < 0.2857c_1$ and $c_1 > 3.5$, or (ii) $e \geq 1$ and $c_1 < 3.5$, or (iii) $e > 1$ and $c_1 = 3.5$. Similarly, the c_1 effect on π_2 is positive if, $e < 0.6667c_1$ and $c_1 > 1.5$; the same effect on t_2 is positive if either, (i) $e > 0.2857c_1$ and $c_1 > 3.5$, or (ii), or (iii) as before. Thus, the positive effects on π_2 require a smaller cost interval compared to the positive effects on t_2.

(b) Team 1 maximizes profits and team 2 maximizes win percentage

We need to use now the profit function (7) and the win percentage function (18).

From the respective first order conditions we obtain:

$$t_1 = \frac{(\sigma - m_1)\, m_2\, (e c_2 (\sigma - m_1) + \theta(-\beta + m_2))}{(\beta\theta + e c_2(-\sigma + m_1))^2} \qquad (32)$$

$$t_2 = \frac{e\theta(-\sigma + m_1)\, m_2^2}{(\beta\theta + e c_2(-\sigma + m_1))^2} \qquad (33)$$

Thus, the win percentages are:

$$w_1 = \frac{e c_2(-\sigma + m_1) + \theta(\beta - m_2)}{\beta\theta + e c_2(-\sigma + m_1)} \qquad (34)$$

$$w_2 = \frac{\theta m_2}{\beta\theta + e c_2(-\sigma + m_1)} \qquad (35)$$

And the profit of team 1 is:

$$\pi_1 = -\frac{(\sigma - m_1)(e c_2(\sigma - m_1) + \theta(-\beta + m_2))^2}{(\beta\theta + e c_2(-\sigma + m_1))^2} \qquad (36)$$

For the following general bounds, $e \geq 1,\ c_2 \succ 0,\ 1 \succ (\beta, \theta, \sigma) \succ 0,\ m_1 \geq 1,\ m_2 \geq \beta,\ m_1 \geq m_2$, it is easy to show that the sum of (32) and (33) is strictly positive.

Notice that in this case, it is unclear whether (32) is larger or smaller than (33). It is also unclear whether the sum of (32) and (33) is larger or smaller than the respective sum of (27) and (28). On the other hand, for plausible parameters, we find (36) > (31).

Table 4b: Some effects from mixed Cournot (team 1 maximizes profits)

$\frac{\partial w_2}{\partial e} = \frac{\theta c_2 (\sigma - m_1) m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	-	$\frac{\partial t_2}{\partial e} = -\frac{\theta (\beta \theta + e c_2 (\sigma - m_1)) (\sigma - m_1) m_2^2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$	-*
$\frac{\partial w_2}{\partial \theta} = \frac{e c_2 (-\sigma + m_1) m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	+	$\frac{\partial t_2}{\partial \theta} = \frac{e (\beta \theta + e c_2 (\sigma - m_1)) (\sigma - m_1) m_2^2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$	+*
$\frac{\partial w_2}{\partial \beta} = -\frac{\theta^2 m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	-	$\frac{\partial t_2}{\partial \beta} = \frac{2 e \theta^2 (\sigma - m_1) m_2^2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$	-
$\frac{\partial w_2}{\partial m_1} = -\frac{e \theta c_2 m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	-	$\frac{\partial t_2}{\partial m_1} = \frac{e \theta (\beta \theta + e c_2 (\sigma - m_1)) m_2^2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$	-*
$\frac{\partial w_2}{\partial m_2} = \frac{\theta}{\beta \theta + e c_2 (-\sigma + m_1)}$	+	$\frac{\partial t_2}{\partial m_2} = \frac{2 e \theta (-\sigma + m_1) m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	+
$\frac{\partial w_2}{\partial \sigma} = \frac{e \theta c_2 m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	+	$\frac{\partial t_2}{\partial \sigma} = -\frac{e \theta (\beta \theta + e c_2 (\sigma - m_1)) m_2^2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$	+*
$\frac{\partial w_2}{\partial c_2} = \frac{e \theta (\sigma - m_1) m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$	-	$\frac{\partial t_2}{\partial c_2} = -\frac{2 e^2 \theta (\sigma - m_1)^2 m_2^2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$	-
$\frac{\partial \pi_1}{\partial e} = \frac{2 \theta c_2 (\sigma - m_1)^2 (e c_2 (-\sigma + m_1) + \theta (\beta - m_2)) m_2}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$			+*
$\frac{\partial \pi_1}{\partial \theta} = \frac{2 e c_2 (\sigma - m_1)^2 m_2 (e c_2 (\sigma - m_1) + \theta (-\beta + m_2))}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$			-*
$\frac{\partial \pi_1}{\partial \beta} = \frac{2 \theta^2 (\sigma - m_1) m_2 (e c_2 (\sigma - m_1) + \theta (-\beta + m_2))}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$			+*
$\frac{\partial \pi_1}{\partial m_1} = \frac{3 e^2 \beta \theta c_2^2 (\sigma - m_1)^2 - e^3 c_2^3 (\sigma - m_1)^3 + \beta \theta^3 (\beta - m_2)^2 - e \theta^2 c_2 (\sigma - m_1)(3 \beta^2 - 2 \beta m_2 - m_2^2)}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$			+*
$\frac{\partial \pi_1}{\partial m_2} = \frac{2 \theta (\sigma - m_1)(e c_2 (-\sigma + m_1) + \theta (\beta - m_2))}{(\beta \theta + e c_2 (-\sigma + m_1))^2}$			-*
$\frac{\partial \pi_1}{\partial \sigma} = \frac{-3 e^2 \beta \theta c_2^2 (\sigma - m_1)^2 + e^3 c_2^3 (\sigma - m_1)^3 - \beta \theta^3 (\beta - m_2)^2 + e \theta^2 c_2 (\sigma - m_1)(3 \beta^2 - 2 \beta m_2 - m_2^2)}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$			-*
$\frac{\partial \pi_1}{\partial c_2} = -\frac{2 e \theta (\sigma - m_1)^2 m_2 (e c_2 (\sigma - m_1) + \theta (-\beta + m_2))}{(\beta \theta + e c_2 (-\sigma + m_1))^3}$			+*

Given the bounds above, we show in Table 4b the sign of derivatives, and under which specific conditions[5] they are positive (those market with a star).

[5] In the win percentage the derivatives were "relatively" easier. The general conditions above were sufficient o check their sign. On the other hand, in the talents, and in profits to team 1, where the

Both e and m_1 affect the win percentage of team 2 negatively, precisely as in case 6(a), Table 4a. Both θ and σ and the own size m_2 affect the win percentage positive while the own costs c_2 affect it negative. Finally, β affects the win percentage negatively now, while in case 6(a) that effect was positive and in case 5, Table 3, could be positive if and only if $c_2 \succ \frac{c_1 m_2}{e m_1}$.

The effects on t_2 are also similar. Moreover, in four cases, these marked effects were conditioned on the specific bounds and values of the parameters.

The effects on π_1 are exactly opposite to the effects on t_2. The effects are similar to those in Table 1, i.e. higher effort and higher market size increase the profit of team 1 and higher values in fixed salary (θ) and bonus (σ) decrease it.

The effects on t_1 are presented in Appendix. Six out of the seven effects are opposite to those on t_2. Only the market size of team 2 has a positive effect on both teams' talents. The effects on t_1 are also similar as in case 6(a), Table 4a, except the effect of β which is positive now, as long as the effort is not higher than 2.99.

7. A further look on the win formulae

So far we used some general or specific values or bounds in parameters, to examine the various effects. In this section we will examine when the win formulae from all models to team 1, i.e. (13), (25), (29) and (34), are larger, equal or lower to the respective formulae to team 2, and also by comparing them to each other.

In order to compare the formulae we need to find simple conditions, satisfying some normalized values and bounds in parameters. Among the nine parameters that appear in all formulae, only e and m_1 exist everywhere. The normalized values $c_2 = 1$, $m_2 = 1$ and the bounds of the remaining seven parameters are given in the first raw of Table 5. Notice that c_2 does not appear in (13), (14), (29) and (30) while the normalized value of $m_2 = 1$ is set in these formulae. The simplest conditions to ensure a perfect competitive balance in all four models are given in the last column, while some numeric values are also provided to ensure that the pair of formulae are ">" or "<".

It is clear that all four formulae can't lead to a perfect competitive balance if the win bonus is strictly positive. Team 1 (team 2) wins more in all formulae for lower (higher) values in θ, σ and c_1, and higher value in γ, irrespectively if teams have almost the same market size and the effort is very close to its upper bound. It is also

derivatives were more complex, these conditions did not provide clear signs. We assumed instead the following specific bounds and values: $c_2 = m_2 = 1, \beta = 0.5, 0 \prec \sigma \leq 0.5, (e, m_1) \geq 1, 0 \prec \theta \leq 1$.

clear that, except formula (34) that can't be larger than (13), all other formulae can be larger or lower than another.

Table 5: Comparison of win formulae

Normalized values and bounds: $c_2 = 1$, $m_2 = 1$ $0 \prec \theta \leq 1,\ 0.1 \leq \sigma \leq 0.5,\ 0 \leq \beta \leq 0.5,\ c_1 \geq 1,\ 0 \prec \gamma \leq 1,\ m_1 \geq 1,\ 1 \leq e \leq 2$								*A simple condition for equality*
	θ	σ	β	c_1	γ	m_1	e	
All formulae = 0.5	*False*[6]							
Team 1 wins more in all formulae	0.5	0.25	0.0625	1.5	0.625	1.0009	1.875	
Team 2 wins more in all formulae	0.9980	0.46875	0.0625	2	0.5	1.0009	1.875	
(13) > (14)	1	0.1	-	-	-	1	1.12	$\theta = e(m_1-\sigma)$
(25) > (26)	-	-	0.25	1	-	1	1.01	$e = \frac{-2c_1+\beta c_1}{\beta-2m_1}, \beta>0$
(29) > (30)	-	-	0.25	1	1	1	1	$e = \frac{-c_1}{\beta\gamma-2\gamma m_1}$
(34) > (35)	1	0.5	0	-	-	1.51	2	$e = \frac{2\theta-\beta\theta}{m_1-\sigma}$
(13) > (25)	1	0.1	0.25	1.85	-	1	2	*Unknown*[7]
(13) < (25)	1	0.1	0.25	1.83	-	1	2	
(13) > (29)	1	0.1	0	2.12	1	1	1	
(13) < (29)	1	0.1	0	2.10	1	1	1	
(13) > (34)	0.5	0.5	0	-	-	1	2	
(13) < (34)	*False*[8]							
(25) > (29)	-	-	0.25	1	1	2.5	1	
(25) < (29)	-	-	0.25	1	1	1	1	
(25) > (34)	1	0.1	0.5	1	-	1	1	
(25) < (34)	1	0.1	0.5	2.67	-	1	1	
(29) > (34)	0.5	0.1	0	3.65	1	1.5	2	
(29) < (34)	0.5	0.1	0	3.67	1	1.5	2	
(29) > (35)	1	0.1	0	1	1	1	1.1111	
(29) < (35)	1	0.5	0	1	0.5	1	1.1111	

6 All can be equal to *0.5* though if $\sigma = 0$, and all other parameters are equal to unit.

7 Despite the fact that we used Mathematica's power function **Reduce,** (that provides all the possible solutions to a set of equations, including those that require specific conditions on parameters), the Mathematica kernel failed to provide solutions for each one of these pairs within 10 minutes of evaluation.

8 It is possible if $\beta = 1$, $\theta = 0.5$, $\sigma = 0.5$, $e = 2$, and $m_1 = 1$.

In the profit maximization model (section 3), the competitive balance is perfect, if $\theta = e(m_1 - \sigma)$ and team 1 wins more if $\theta < e(m_1 - \sigma)$. For instance, for given, θ, m_1 and σ-values, if effort is higher than *1.11*, team 1 wins more.

In the win maximization model (section 4), the competitive balance is perfect, if $e = \frac{-2c_1+\beta c_1}{\beta-2m_1}, \beta>0$ and team 1 wins more if $e > \frac{-2c_1+\beta c_1}{\beta-2m_1}$. For example, for $\beta = 0.25$, and $m_1 = c_1 = e = 1$, the competitive balance is perfect. Given $m_1 = e = 1$, for $c_1 > 1$, team 2 wins more, while for higher values in m_1, and/or e team 1 wins more, if $c_1 = 1$. Notice that for $\beta = 0$, both (25) and (26) are indeterminate.

In the first mixed model (section 6(a)), the competitive balance is perfect, if $e = \frac{-c_1}{\beta\gamma-2\gamma m_1}$ and team 1 wins more if $e > \frac{-c_1}{\beta\gamma-2\gamma m_1}$. For instance, for $m_1 = e = \gamma = 1$ and $\beta = 0$, the competitive balance is perfect even if $c_1 = 2$. The competitive balance is still perfect for $m_1 = 2$, $e = 1$ and $\beta = 0$, even if $c_1 = 4$ and $\gamma = 1$.

In the second mixed model (section 6(b)), the competitive balance is perfect, if $e = \frac{2\theta-\beta\theta}{m_1-\sigma}$ and team 1 wins more if $e > \frac{2\theta-\beta\theta}{m_1-\sigma}$. For instance, for $\theta = 1$, $\sigma = 0.5$, $\beta = 0$ and $e = 2$, team 1 wins more if its market size is just slightly more than 50% higher, while when both teams have the same market size and $\theta = \sigma = 0.5$, $\beta = 0$ and $e = 2$, the competitive balance is perfect.

Thus, in our four models, the interaction of teams' various parameters, does not necessarily support Dietl et al (2009) findings, because the competitive balance can be perfect, even if the larger team pays both high fixed salary and bonus, or has higher costs than the smaller team, as long as its market size is larger and/or its effort is higher.

When we compare all the win formulae for team 1 pair wise, i.e. (13), (25), (29) and (34), we conclude that (29) is preferred to all others (in bald), if team 1 manages to keep its costs at most *2.11* times higher compared to team 2, or even if it has the same market size as team 2. Since (25) is preferred to (13), and to (34), (again if team 1 manages to keep its costs at low levels) team 1 would like to maximize its win percentage and not its profit.

Finally, when we compare the two mixed cases for team 1, (last two rows), it is not clear whether (29) is higher or lower than (35). When team 2 maximizes profits and team 1 win percentage, the win percentage to team 1 (i.e. (29)), is higher than (35), (i.e. the reverse case), provided that the bonus is low and the wages to team 2 high. On the other hand, when the bonus is high and the wages to team 2 are low, and for the same values in other parameters, team 2 wins more than team 1.

8. Comparison of the four models

In this section, precisely as in the previous one, instead of using some ad-hoc values, we use a simple non-linear program to implicitly find the "optimal" values in parameters. We need to maximize each one of these eight formulae, first for team 1 and second for team 2, given some constraints on the remaining ones. We formulated therefore the following non-linear program.

$Max\ w^i_{1,2}$

$s.t.\ w^i_{1,2} < 0.8$

$0.35 < w^j_{1,2} < 0.8,\ i \neq j$

Bounds

The objective function maximizes the win percentage for each one of these four formulae. In order to exclude the extreme solutions of a win percentage of *100%*, the objective function's win percentage has an upper bound of *0.8* (i.e. a very high imbalance). We also assume that, the other three formulae for the same team (from the respective models) should have an upper and a lower bound of *0.8* and *0.35* respectively. We used the same general bounds and the two normalized values as in the previous section.

The estimates are shown in Tables 6a and 6b. Some parameters are in bald while others are in plain text. The set of bald parameters are the implicitly "optimal" values that appear in the respective win percentage objective function, while the set of plain parameters are found from the constraints and the bounds of the other three formulae[9]. The bald values in the win percentage, the talents and the profits (both normalized), are the respective "optimal" values which are consistent with the formulae of the same model. Similarly, the plain text values are obtained when the "optimal" parameters are set in the respective formulae of the other three models.

For instance, when team 1 maximizes, the bounds (from both the bald and the plain parameters) are:

$$0.36 < \beta \leq 0.5,\ 0.04 \prec \sigma \leq 0.44,\ 0.52 \prec \theta \leq 0.84,\ 0.38 \prec \gamma \leq 0.97,$$
$$1.41 \prec m_1 \leq 2.5,\ 1.05 \prec e \leq 1.81,\ 2.33 \prec c_1 \leq 2.76$$

Moreover, in order to simplify the comparison of all these "optimal" parameters, we will concentrate on the bald parameters of both Tables. The following general conclusions are drawn:

[9] As was mentioned earlier, the normalized value $c_2 = 1$, is excluded when γ is included and also in formulae (13) and (14).

First, as expected, the highest imbalances seem to appear when each team maximizes anyone of its four formulae. It seems also that the competitive balance is higher in Table 6b, if we concentrate on the plain text parameters (off-diagonal).

Table 6a: Summary results (Team 1 maximizes the win percentage)

c_2= 1, m_2= 1 *Bounds:* $0 \prec \theta \le 1,\ 0.1 \le \sigma \le 0.5,\ 0 \le \beta \le 0.5,\ c_1 \ge 1,\ 0 \prec \gamma \le 1,\ m_1 \ge 1,\ 1 \le e \le 2$								
	Win percentage to team 1 and 2 (raw 1 & 2), when each one of the four formulas is maximized				*Talents to team 1 and 2 (row 1 & 2) and Profits to team 1 and 2 (row 3 & 4) according to the following four formulas*			
	(13) *π-max*	**(25)** *win-max*	**(29)** *π-max, team 2*	**(34)** *π-max, team 1*	**(9)** **(10)** **(11)** **(12)**	***(23)*** ***(24)*** **–** **–**	***(27)*** ***(28)*** **–** ***(31)***	***(32)*** ***(33)*** ***(36)*** **–**
***θ* = .5176,** ***σ* = .4405,** *β* = .4248, c_1= 2.759, *γ* = .7831, **m_1= 2.222,** ***e* = 1.165**	**0.8** **0.2**	0.6 0.4	.6442 .3558	.7745 .2255	**55** **16** **101** **4**	43 33 - -	45 29 - 13	60 20 101 -
θ = .7422, *σ* = .0738, ***β* = .3578,** **c_1= 2.511,** *γ* = .3808, **m_1= 2.496,** ***e* = 1.055**	.7749 .2251	**0.8** **0.2**	.3777 .6222	.7369 .2631	57 17 101 5	**70** **19** - -	36 62 - 39	63 24 101 -
θ = .8455, *σ* = .3487, ***β* = .4396,** **c_1= 2.329,** ***γ* = .9663,** **m_1=1.415,** ***e* = 1.813**	.6957 .3043	.6923 .3077	**0.8** **0.2**	.6332 .3668	27 21 100 9	33 27 - -	**37** **17** **-** **4**	29 31 100 -
***θ* = .6220,** ***σ* = .0389,** ***β* = .5,** c_1= 2.403, *γ* = .9273, **m_1= 1.647,** ***e* = 1.547**	0.8 0.2	.6819 .3181	.7573 .2427	**.7778** **.2222**	41 16 101 4	37 27 - -	40 20 - 6	**45** **20** **101** **-**

Table 6b: Summary results (Team 2 maximizes the win percentage)

$c_2 = 1$, $m_2 = 1$ *Bounds:* $0 \prec \theta \le 1,\ 0.1 \le \sigma \le 0.5,\ 0 \le \beta \le 0.5,\ c_1 \ge 1,\ 0 \prec \gamma \le 1,\ m_1 \ge 1,\ 1 \le e \le 2$								
	Win percentage to team 1 and 2 (raw 1 & 2), when each one of the four formulas is maximized				*Talents to team 1 and 2 (row 1 & 2) and Profits to team 1 and 2 (row 3 & 4) according to the following four formulas*			
	(14) *π-max*	**(26)** *win-max*	**(30)** *π-max, team 2*	**(35)** *π-max, team 1*	**(9)** **(10)** **(11)** **(12)**	***(23)*** ***(24)*** **–** **–**	***(27)*** ***(28)*** **–** ***(31)***	***(32)*** ***(33)*** ***(36)*** **–**
$\boldsymbol{\theta = .9795}$,	**.4286**	.3946	.5453	0.2	**18**	27	34	12
$\boldsymbol{\sigma = .3021}$,	**.5714**	.6054	.4547	0.8	**24**	42	29	48
$\beta = .5$,					**100**	-	-	100
$c_1 = 1.206$,					**33**	-	21	-
$\gamma = .8604$,								
$\boldsymbol{m_1 = 1.023}$,								
$\boldsymbol{e = 1.019}$								
$\theta = .8367$,	.6453	**0.2**	.4537	.5614	27	**8**	16	28
$\sigma = .1335$,	.3547	**0.8**	.5463	.4386	23	**51**	31	35
$\boldsymbol{\beta = .4608}$,					100	-	-	100
$\boldsymbol{c_1 = 2.513}$,					13	-	30	-
$\gamma = .8119$,								
$\boldsymbol{m_1 = 1.109}$,								
$\boldsymbol{e = 1.560}$								
$\theta = .8470$,	.6499	.2282	**0.2**	.5741	23	8	**7**	25
$\sigma = .1814$,	.3501	.7718	**0.8**	.4259	23	48	**50**	34
$\boldsymbol{\beta = .4916}$,					100	-	**-**	100
$\boldsymbol{c_1 = 2.737}$,					12	-	**64**	-
$\boldsymbol{\gamma = .3175}$,								
$\boldsymbol{m_1 = 1.042}$,								
$\boldsymbol{e = 1.826}$								
$\boldsymbol{\theta = .9844}$,	.4560	.2862	.4466	**0.2**	20	20	29	13
$\boldsymbol{\sigma = .4784}$,	.5440	.7138	.5534	**0.8**	25	50	37	54
$\boldsymbol{\beta = .4118}$,					100	-	-	**100**
$c_1 = 1.690$,					30	-	31	**-**
$\gamma = .6705$,								
$\boldsymbol{m_1 = 1.287}$,								
$\boldsymbol{e = 1.021}$								

Second, the profits to team 1 are always higher and approximately 100, irrespectively if team 1 maximizes, either (13) or (34) or team 2 maximizes either (14) or (35). The maximum profit for team 2 (i.e. *64*), is obtained when that team maximizes (30). The sum of profits from (11) and (12) in Table 6b is higher than the same sum in Table 6a.

Third, the number of talents and its distribution varies. When team 1 maximizes its formulae, the range is higher. The normalized upper value is *98*, when formula (25) is maximized and the "optimal" values are set into formulae (27) and (28). In fact, in this case, team 2 maximizes its talents too. The lower value is *42*, when formula (14) is maximized. The lower and higher values for team 1 are *7* and *70* respectively, while the respective values for team 2 are *16* and *62*. Notice also two interesting features: (i) when team 2 maximizes (30), (and achieves the maximum profit, *64*), team 1 has its minimum value in talents, 7; (ii) when team 1 maximizes (25) it maximizes its talents as well, *70*. In case (ii), if the "optimal" values are set into (28) team 2 achieves its maximum number of talents, *62*; and if are set in (29), team 2 wins more than team 1 in Table 6a, (i.e. *62%* versus *38%*).

Thus, in general, the "optimal" parameter values from Table 6b are more appropriate to maximize the balance of teams in terms of talents, the competitive balance and total profits.

A more detailed investigation of the "optimal" parameters reveals the following:

Two parameters, m_1 and e appear in all formulae. As expected, when team 1 maximizes, m_1 is always higher. On the other hand the values of e are not necessarily higher, when team 1 maximizes.

The parameter β is included in six formulae. The bald values in both tables are also rather similar, with bounds around *0.4* and *0.5*.

The fixed wage (θ) and the win bonus (σ) appear in four formulae. While, as expected, the sum of θ and σ is always lower when team 1 maximizes, the bald value of σ is lower in (14), compared to (13).

The cost parameter c_1 is included in four formulae. In both win max formulae (25) and (26), the value is the same, and as expected, a bit lower when formula (29) is maximized and a bit higher when formula (30) is maximized.

Finally, the parameter γ (the wages to team 2) is included in two formulae. As expected, its value is lower when team 2 maximizes (30).

Thus, it is clear that team 1 would like to have a larger market size compared to what team 2 would like them to have. The minimum market size team 1 would like to have is *40%* higher than the normalized market size of team 2, while team 2 would prefer that to be at most *29%* higher. Effort, in relation to market size, makes no difference. This is clearly shown in the maximization of (25) and (26). When (26) is maximized, the "optimal" value of effort is much higher compared to when (25) is maximized, (i.e. *1.56* versus *1.05*). Moreover, that is not sufficient to make team 1 winning more, because the market size of team 1 is much lower compared to when (25) is maximized (i.e. *1.11* versus *2.50*). On the contrary, given the fact that the other two bald parameters are quite similar in both Tables, the win percentages shift from

0.8 - 0.2 to *0.2 – 0.8*! A similar finding is when we compare (13) and (34). Despite the fact that the win bonus is higher and the effort is lower in (13), the larger market size makes the win percentage *0.8* in (13) and *0.78* in (34). This is also consistent with the previous findings in Table 5, where (13) is never lower than (34).

It is also clear that team 1 would like to pay a lower fixed salary (θ), compared to what team 2 would like them to pay. On the other hand, when both teams maximize profits (and the relevant formulae are (13) and (14)), team 1 is willing to pay higher bonus (σ) to its talents, compared to what team 2 would like them to pay. The levels of bonus are reverse when team 1 maximizes profits and team 2 maximizes win percentage (and the relevant formulae are (34) and (35)).

It is also clear that while team 2 would like to pay lower wages (γ) to their own talents, (when team 2 maximized profits and team 1 maximizes win percentage), team 1 would prefer them to pay three times higher.

The higher values of c_1, compared to the normalized value of $c_2 = 1$, are unexpected. Their effect is less important when other parameters are favourable to team 1. For instance, the maximization of (29) yields $w_1 = 0.8$ and $w_2 = 0.2$, despite the fact that c_1 is 133% higher than c_2.

9. **Conclusions**

It is rather difficult to draw clear-cut conclusions, to the competitive balance, profits and talents from all these models. We concentrate only on the competitive balance and on the clear derivatives of profits and talents, and illustrate with a big team, like Real Madrid and a smaller one, like Seville.

Regarding the partial derivatives from all four models, the competitive balance improves when: (i) Real Madrid reduces its market size; (ii) the effort of its talents decreases; (iii) the fixed salary, the win bonus and the other cost parameter c_1 increase; and (iv) when Seville decreases its wages, its cost parameter c_2 , and increases its market size.

Moreover, the strength of these derivatives varies. It is not clear which of these parameters is the most crucial. In some models it is sufficient when one of these parameters is above or below some critical values for the competitive balance to decrease. In the win-maximization model both effort and c_1 need to take high and minimum values respectively to make Real Madrid winning more.

A high win bonus that improves the effort of Real Madrid players significantly will decrease the competitive balance. On the other hand, when the introduction of a win bonus does not improve the effort of the players, the competitive balance will improve if Real Madrid has introduced it and will decrease if Seville has introduced it.

Regarding the partial derivatives from all four models on talents, t_1 and t_2, the signs, in most cases, are similar to those on w_1 and w_2. It seems though that the market size of Real Madrid is more important than, for instance, the effort of its talents, or the costs c_1, in order to increase the number of talents, at least for themselves. In case the market size of Real Madrid is not much higher than Seville, high effort from Real Madrid talents and simultaneously low win bonus paid to them are required instead to increase t_1.

Finally, the effects on profits π_1 and π_2 are also similar to those on w_1 and w_2. Moreover, the profits Seville are never higher, even if Real Madrid pays a high fixed salary and bonus, or if Seville pays very low wages. A slightly higher market size of Real Madrid and a slightly higher effort from its talents are sufficient to ensure that $\pi_1 > \pi_2$.

Appendix

Case 6(b): The talent derivatives for team 1 and the conditions under which they are positive.

$$\frac{\partial t_1}{\partial e} = \frac{c_2(\sigma - m_1)^2(e c_2(\sigma - m_1) - \theta(\beta - 2m_2))m_2}{(\beta\theta + e c_2(-\sigma + m_1))^3},$$
$$\frac{\partial t_1}{\partial \beta} = \frac{\theta(\sigma - m_1)(e c_2(-\sigma + m_1) + \theta(\beta - 2m_2))m_2}{(\beta\theta + e c_2(-\sigma + m_1))^3},$$
$$\frac{\partial t_1}{\partial c_2} = \frac{e(\sigma - m_1)^2(e c_2(\sigma - m_1) - \theta(\beta - 2m_2))m_2}{(\beta\theta + e c_2(-\sigma + m_1))^3}$$

All three above are positive for the same condition below. The condition requires an upper bound of effort equal to 2.99.

$$\left(1 \le m_1 \le 1.5\ \&\ 0.333333(-2\sigma + 2m_1) < \theta \le 1\ \&\ 1 \le e < -\frac{3\theta}{2\sigma - 2m_1}\ \|\ 1.5 < m_1 < 2\ \&\ 0.5(-3 + 2m_1) < \sigma \le 0.5\ \&\ 0.333333(-2\sigma + 2m_1) < \theta \le 1\ \&\ 1 \le e < -\frac{3\theta}{2\sigma - 2m_1}\right)$$

$$\frac{\partial t_1}{\partial m_1} = \frac{\theta m_2(\beta\theta(\beta - m_2) - e c_2(\sigma - m_1)(\beta + m_2))}{(\beta\theta + e c_2(-\sigma + m_1))^3} > 0$$

$$\frac{\partial t_1}{\partial \theta} = -\frac{(\sigma - m_1)m_2(\beta\theta(-\beta + m_2) + e c_2(\sigma - m_1)(\beta + m_2))}{(\beta\theta + e c_2(-\sigma + m_1))^3} < 0$$

$$\frac{\partial t_1}{\partial \sigma} = \frac{\theta m_2(\beta\theta(-\beta + m_2) + e c_2(\sigma - m_1)(\beta + m_2))}{(\beta\theta + e c_2(-\sigma + m_1))^3} < 0$$

$$\frac{\partial t_1}{\partial m_2} = \frac{(\sigma - m_1)(e c_2(\sigma - m_1) - \theta(\beta - 2m_2))}{(\beta\theta + e c_2(-\sigma + m_1))^2}$$

It is positive for the following condition. This condition does not require an upper bound of effort.

$$1 \le m_1 \le 1.5\ \&\ \left(0 < \theta < 0.3333333(-2\sigma + 2m_1)\ \&\ e \ge 1\ \|\ \theta == 0.33333333(-2\sigma + 2m_1)\ \&\ e > 1\ \|\ 0.33333333(-2\sigma + 2m_1) < \theta \le 1\ \&\ e > -\frac{3\theta}{2\sigma - 2m_1}\right)\ \|\ 1.5 < m_1 \le 2\ \&$$
$$\left(0 < \sigma < 0.5(-3 + 2m_1)\ \&\ 0 < \theta \le 1\ \&\ e \ge 1\ \|\ \sigma == 0.5(-3 + 2m_1)\ \&\ (0 < \theta < 1\ \&\ e \ge 1\ \|\ \theta == 1\ \&\ e > 1)\ \|\ 0.5(-3 + 2m_1) < \sigma \le 0.5\ \&\ \left(0 < \theta < 0.33333333(-2\sigma + 2m_1)\ \&\ e \ge 1\ \|\ \theta == 0.333333333(-2\sigma + 2m_1)\ \&\ e > 1\ \|\ 0.333333333(-2\sigma + 2m_1) < \theta \le 1\ \&\ e > -\frac{3\theta}{2\sigma - 2m_1}\right)\ \|\ m_1 > 2\right)$$

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